

# Mathematics II

(English course)

Second semester, 2012/2013

## Exercises (7)

1. Find and classify the critical points of the following functions:

(a)  $f(x, y) = \frac{xy}{x^2+y^2}$ ,  $(x, y) \in \mathbb{R}^2 \setminus \{(0, 0)\}$ ;

(b)  $f(x, y) = \frac{(x+1)y}{x^4+2y^4}$ ,  $(x, y) \in \mathbb{R}^2 \setminus \{(0, 0)\}$ ;

(c)  $f(x, y) = (x-1)(y-1)(x+y-1)$ ,  $(x, y) \in \mathbb{R}^2$ ;

(d)  $f(x, y) = (y-x^2)(y-1)$ ,  $(x, y) \in \mathbb{R}^2$ ;

(e)  $f(x, y) = xy(xy-2)$ ,  $(x, y) \in \mathbb{R}^2$ ;

(f)  $f(x, y, z) = (xy+z^2)e^{-x^2-y^2-z^2}$ ,  $(x, y, z) \in \mathbb{R}^3$ .

2. Consider the function  $f : \mathbb{R}^2 \setminus \{(0, 0)\} \mapsto \mathbb{R}$ , defined as

$$f(x, y) = \frac{y}{x^2 + y^2}.$$

Does  $f$  admit any extremum? Why?

3. Find the extrema of the following functions on the corresponding sets.

(a)  $f(x, y) = x^2 + xy - y^2$ ,  $A = \{(x, y) : x^2 + y^2 = 1\}$ .

(b)  $f(x, y) = x + 2y$ ,  $A = \{(x, y) : 4x^2 + 6xy + 3y^2 = 1\}$ .

(c)  $f(x, y) = x^2 - xy + y^2$ ,  $A = \{(x, y) : (x-y)^2 + (x+y)^2 = 4\}$ .

(d)  $f(x, y, z) = x + 2y - z$ ,  
 $A = \{(x, y, z) : (x-1)^2 + (y+1)^2 + z^2 = 4\}$ .

(e)  $f(x, y, z) = (x+2y)^2 - (y+z)^2 + z^2$ ,  
 $A = \{(x, y, z) : x^2 + y^2 + z^2 = 9\}$ .

(f)  $f(x, y, z) = xy + yz$ ,  
 $A = \{(x, y, z) : (x-4)^2 + y^2 + z^2 = 4 \wedge x + y + z = 1\}$ .

(g)  $f(x, y, z) = x^2 + 2y^2 + 3z^2$ ,  
 $A = \{(x, y, z) : x^2 + y^2 + z^2 = 1, x + 2y + 3z = 0\}$ .

(h)  $f(x, y, z) = xyz$ ,  
 $A = \{(x, y, z) : x^2 + y^2 + z^2 = 1 \wedge xy + yz + zx = 1\}$ .

4. Find the points of the set  $\{(x, y, z) : xyz = 1, x^2 + y^2 + 2z^2 = 4\}$  that lie farther and closer to the origin.
5. Find the shortest segment of a straight line connecting the parabola  $y = x^2$  to the straight line  $x - y = 2$ .
6. Find the extrema of the following functions on the corresponding sets.
- (a)  $f(x, y) = x^2 + y^2 - x - 3$ ,  $A = \{(x, y) : 2x^2 + y^2 \leq 1\}$
- (b)  $f(x, y) = x^2 + xy - y^2$ ,  $A = \{(x, y) : x^2 + y^2 \leq 1\}$ .
- (c)  $f(x, y) = x^2 - xy + y^2$ ,  $A = \{(x, y) : (x - y)^2 + (x + y)^2 \leq 4\}$ .
- (d)  $f(x, y, z) = xy + yz$ ,  
 $A = \{(x, y, z) : (x - 4)^2 + y^2 + z^2 \leq 4 \wedge x + y + z \geq 1\}$ .
- (e)  $f(x, y, z) = x^2 + 2y^2 + 3z^2$ ,  
 $A = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1, x + 2y + 3z \leq 0\}$ .
- (f)  $f(x, y, z) = xyz$ ,  
 $A = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1 \wedge xy + yz + zx = 1\}$ .